Assignment 5.

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Q(2) - b.

Step1. Show that Efficient Recruiting is in NP.

Given a set of k applicants as a "guess", the cost of checking whether all the n sports are covered is $O(nk)$. For example, This "guess" can be represented by 0/1 table of size $n \times k$ of sports x Counselors. We need to verify that there is at least one '1' in each row.

	C_1	C_2	...	C_k
S_1	1	0		1
S_2	0	1		0
⋮		⋮		
S_n	0	1		1

Step2. Show that a known NPC problem is polynomially reducible to Efficient Recruiting.

Important Side Note: The whole point of NP-completeness is that any one of these problems can be polynomially reduced to the problem X, in question, if the latter is indeed NPC. However, the reduction to a given problem X can be substantially easier starting from some problems than from others. Hence, one should try to choose the "right" one to use in a particular reduction.

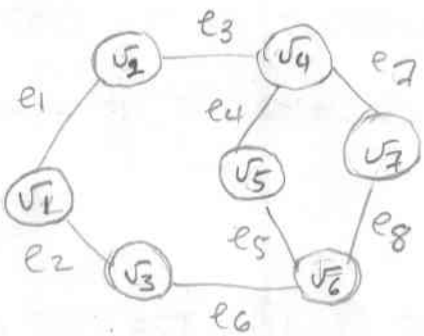
For Efficient Recruiting, a natural analogue can be the Vertex Cover problem, which is a known NPC problem.

(V.C)

Step 2.1 Show That we can map an instance of Vertex Cover to an instance of Efficient Recruiting in polynomial-time.
(E.R.).

Given a graph $G = (V, E)$ and an integer k as an instance of V.C., we construct a sport S_e for each edge $e \in E$ and a Counselor C_v for each vertex $v \in V$. We set a counselor C_v to be qualified in Sport S_e iff e is incident to node v.
This information can be represented by a 0/1 table of size $|V| \times |E|$ such as The one described previously.

Ex.



$\Longrightarrow$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
S_1	1	1	0	0	0	0	0
S_2	1	0	1	0	0	0	0
S_3	0	1	0	1	0	0	0
S_4	0	0	0	1	1	0	0
S_5	0	0	0	0	1	1	0
S_6	0	0	1	0	0	1	0
S_7	0	0	0	1	0	0	1
S_8	0	0	0	0	0	1	1

The cost of This transformation is polynomial, specifically $O(mn)$ where n is The number of sports & m is the number of counselor (applicants).

Step 2.2. Show That the constructed instance of E.R. has k counselors that cover all n sports If and only if The given instance of V.C. has an a vertex cover of size k.

That is, YES instance of E.R. implies YES instance of V.C and YES instance of V.C. implies YES instance of E.R.

$\longrightarrow$ Cont.

(1) If the constructed E.R. has a set of k counselors such that all n sport are covered, This implies That each sport (i.e. edge) is covered by at least one counselor (i.e. node) indicated by at least one '1' in each row of table T. Hence, The corresponding set of k vertices in G have the property that each edge has an end in at least one of them; So, they define a vertex cover of size k.

(2) Conversely, if there is a vertex cover of size k in the given instance G, Then The corresponding counselors in The constructed E.R. instance has the property that each sport is covered by at least one of them.

⇒ Thus, G has a vertex cover of size at most k iff the constructed instance of E.R. can be solved with at most k counselors.

To sum up, If we could solve E.R. in polynomial-time, we could also solve V.C. in polynomial-time. So, E.R. is NP-Complete.