SYDE 423 - Fall 2008. Assignment 1.

Assigned Wednesday Sept 17. Due Monday September 29.

1. Solve the following questions from the textbook [L]. Questions 8 and 9 in exercises 2.1, and question 7 in exercises 2.4. **Hint:** There are some hints at the back of the book (you may have a look!).

2. Consider the problem of finding the greatest common divisor of two positive integers m and n, $\gcd(m, n)$. Assume that $m \geq n \geq 0$. Consider two algorithms for computing $\gcd(m, n)$. One is Euclid's algorithm that takes advantage of the equality $\gcd(m, n) = \gcd(n, m \bmod n)$ by repeatedly applying it. The other is the sequential check algorithm that starts by checking n and proceeds by decrementing it sequentially, in search for the solution.

(a) Analyze the worst-case run time of Euclid's algorithm. Give the steps of your analysis and a tight asymptotic bound. **Hint:** $(m \bmod n) \leq m/2$.

(b) Analyze the worst-case run time of the sequential check algorithm. Give the steps of your analysis and a tight asymptotic bound.

(c) ***Programming Question:*** Implement each of the above algorithms. For Euclid's algorithm, worst case inputs happen to be consecutive elements of the Fibonacci sequence. Implement an efficient algorithm for generating Fibonacci numbers and use them as input for Euclid's algorithm. Record the runtime corresponding to each input sample. For the sequential checking algorithm, you can use the generated Fibonacci numbers as input or design other input of your choice and record the runtime. Present your results and comment on whether they agree with the theoretical analysis. Check the progamming guidelines on the course webpage. Submit your source code by e-mail before Monday's lecture.

3. Consider the *stable marriage* problem and the algorithm given in the handout for Lect. 2. Suppose that the following instance is given, where for each woman $w \in \mathcal{W}$ and each man $m \in \mathcal{M}$, the preferences are listed in descreasing order as follows.

$\mathcal{W}$	m_1	m_2	m_3
Vallerie	Jason	Bill	Mark
Monica	Bill	Mark	Jason
Heather	Mark	Jason	Bill

$\mathcal{M}$	w_1	w_2	w_3
Jason	Monica	Heather	Vallerie
Bill	Heather	Vallerie	Monica
Mark	Vallerie	Monica	Heather

(a) Find the solution computed by the version of the algorithm in which women propose.

(b) Find the solution computed by the version of the algorithm in which men propose.

(c) Can you identify a different solution that neither versions identify? Comment on the solutions.

4. Question 6, Ch. 2. in the book [KT]. The question is summarized here as follows. Given an array A of n integers, $A[1], A[2], \ldots, A[n]$, the problem is to compute an $n \times n$ array B in which $B[i, j] = A[i] + A[i + 1] + \cdots + A[j]$, for $i < j$. The values $B[i, j]$ for $i \geq j$ do not matter. The algorithm is as follows.

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for i = 1 to n do
  for j = i + 1 to n do
    compute B[i, j] as sum of A[i] through A[j]
  end for
end for
```

(a) Give a tight bound on the running time of the algorithm.

(b) The algorithm contains unnecessary computations. Give a different algorithm with a better asymptotic running time.