SYDE 423 - Fall 2008. Assignment 2.

Assigned Monday Sept 29. Due Monday October 6.

1. From the textbook, solve questions 1 and 5 in 3.3.

2. For the stable marriage problem in handout 1, solve the following questions:

(a) What is the maximum number of iterations $c(n)$ of the while loop?

(b) To keep the run time bound of the algorithm $t(n) = \Theta(c(n))$, each of the operations associated with the while loop should take a constant time. Outside the loop, operations may be $O(c(n))$. This requires adequate use of data structures. Suppose that we associate a unique number in $\{1, \ldots, n\}$ to each man and each woman. That is, we have $m, w \in \{1, \ldots, n\}$. Hence, the input can be completely specified by two two-dimensional arrays MPref and WPref where MPref$[m, i]$ denotes the i-th woman on m's preference list and WPref$[w, i]$ denotes the i-th man on w's preference list. To allow each of the following loop-operations to take $\Theta(1)$ time, describe the required data structure, the operations that the algorithm will perform on it, including initializations, and specify the run time bounds of these operations.

i. Identify a free woman (the condition of the while-loop)

ii. For a woman w, identify the highest-ranked man to whom w has not yet proposed.

iii. Determine if a man m is currently matched with a woman and identify her.

iv. For a man m and two women w and w', determine the one preferred by m.

(c) If we are to use the exhaustive search approach for the stable marriage problem, what would be the size of the search-space (if we make use of the first condition)?

3. Suppose that a graph $G = (V, E)$ is given by its adjacency list representation. Design an efficient $O(|V| + |E|)$ algorithm for finding out if G is connected, using either one of the following traversals (choose any one). Make use of suitable data structures.

- Breadth-first traversal, which constructs a BF search-tree rooted at the starting node.
- Depth-first traversal, which constructs DF search-tree rooted at the starting node.

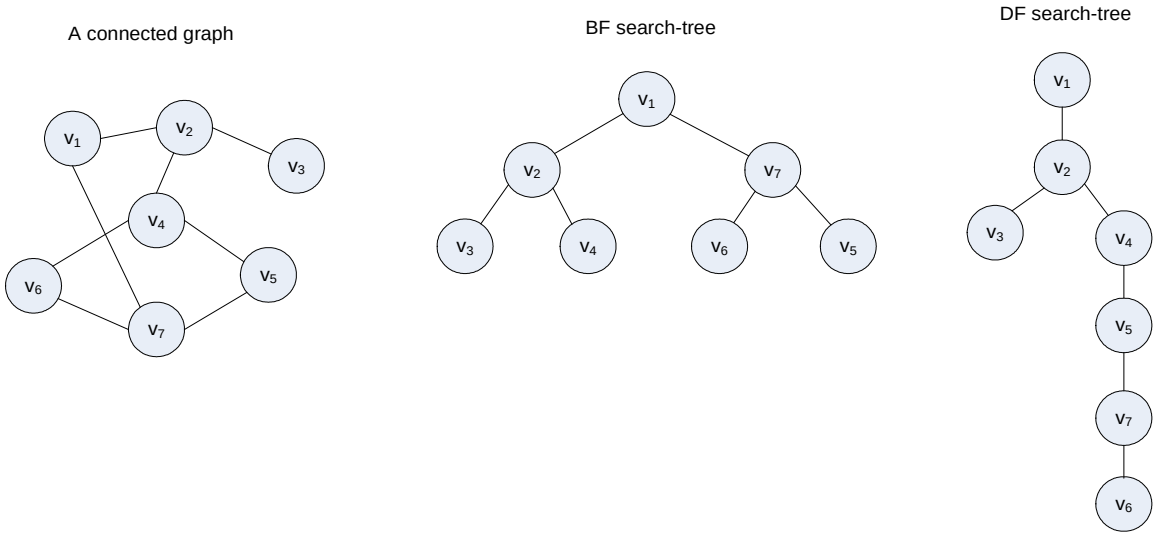

Figure 1: An example of a connected graph and its corresponding search trees rooted at V_1.

3. Illustrate the following operations on a binary min-heap, by sketching the tree and its corresponding representation as an array.

- Insert the list of numbers $12, 6, 7, 15, 4, 3, 9$. Then delete the minimum element twice.