SYDE 423 - Fall 2008. Assignment 5.

Assigned Monday Nov 17. Due Monday Nov 24.

1. From the textbook, solve the following:

- Question 10 in Exercises 11.3
- Question 5, 7 in Exercises 12.1

2. Questions 2 and 3 in Ch 8 of the textbook by Kleinberg and Tardos, which are summarized below.

(a) Stores often try to analyze the behavior of their customers based on purchasing data. Suppose that such data can viewed as a two-dimensional table T where the rows correspond to the customers and columns correspond to the products. An element $T[i, j]$ represents the quantity of product j that has been bought by customer i. Sometimes, it is of interest to find a "diverse" set of customers, for market research purposes. Let a subset S of the customers be called *diverse* if no two of the customers in S have ever bought the same product. It is required to find a diverse subset of customers that is as large as possible, as a target pool for market analysis.

Let the decision version of this problem be called Diverse Subset, which can be stated as follows. Given an $m \times n$ table T as defined above, and a numbers $k \leq m$, decide whether there is a diverse subset of at least k customers.

Show that Diverse Subset is NP-complete.

(b) Suppose that in organizing sports events such as summer camps, the following problem arises. It is required that the camp has at least one counselor who is skilled at each of the n sports covered by the camp. Job applications from m potential counselors have been received, where for each of the n sports, there is some subset of the m applicants who are qualified for it. The problem is, for a given number $k < m$, is it possible to hire at most k counselors such that at least one counselor is qualified in each of the n sports? Let this problem be called Efficient Recruiting.

Show that Efficient Rectruiting is NP-complete.

Please note that in these two questions, a proof of polynomial reducibility is required. An example of such proof is the one presented in the textbook by Levitin (p 399) for reducing the Hamiltonian Circuit problem to the decision version of the Traveling Salseman problem.