Assign. 5

Q (2) (a)

Step 1 : Show that Diverse Subset is in NP

Given a set of k customers, we can check if they constitute a diverse subset by checking whether each of C^k_2 customers ever bought any of n products in $\Theta(k^2 n)$ time, which is polynomial in k and n. Since $k \leq m$, the time efficiency is also bounded by $\Theta(m^2 n)$, where m is the total number of customers and n is the total number of products.

Step 2 : Show that Indepent Set $\leq_p$ Diverse Subset, where Independent set is a known NP-complete problem.

First, show that an instance of independent set can be transformed into an instance of Diverse Subset in polynomial-time:

Given a graph $G = (V, E)$ and a number k, we set a customer for each node in V and a product for each edge in E. This can be represented by a $|V| \times |E|$ table T which indicates that a customer v bought product e if edge $e \in E$ is incident to node $v \in V$. (Alternatively, the information in this table can also be represented by a $|V| \times |V|$ adjacency table).

We then ask whether this table contains a diverse subset of size k.

Second, we show that this table (instance of D.S. problem) has a diverse subset of size k iff G (instance of Independent Set) has an independent set of size k.

① If there is a diverse subset of customers of size k (YES inst of D.S), then the corresponding set of nodes has the property that no two are connected by an edge, based on the construction of T from G. So, it is an independent set of size k.

② Conversely, If G has an independent set of size k (YES instance of I.S.), then by construction of T the corresponding set of customers represented in T has the property that no two bought the same product, so, it is a diverse subset.