

SYDE 423 - Fall 2008. Optional Assignment 1.

Assigned Friday Oct 24. Due Friday October 31.

Any of the questions below may be used for improvement of marks. Please note that giving correct and clear solutions is a requirement for scoring marks on optional assignments.

1. Compare the growth rate of $\ln^p n$ with n^q , for any $p, q > 0$ (including $0 < q < 1$).
2. Suppose that you are given a sorted list of n elements followed by $f(n)$ unsorted elements. How large can $f(n)$ be for the entire list to be sortable in $O(n)$ time? (Weiss, 2006)
3. Solve question 3 in Exercises 3.2.
4. Consider the following definition of the Fibonacci numbers:

$$\begin{aligned} F_{2n} &= (F_n + F_{n-1})^2 - F_{n-1}^2 \\ F_{2n+1} &= (F_n + F_{n-1})^2 + F_n^2 \end{aligned}$$

where $F_0 = 0$ and $F_1 = 1$.

- (a) Give a divide-and-conquer algorithm for computing F_n for $n \geq 0$ based on the above definition.
- (b) Give a tight bound on the run time of the algorithm based on the number of bit operations (essentially the multiplication). Write the recurrence for the run time and solve it using the master theorem, making use of the fact that $F_n \leq 2^n$ and supposing that the fast integer/bit multiplication algorithm for large numbers is being used, which takes time $\Theta(n^{1.59})$ where n is the number of bits.