Handout for Lecture 2.
The Stable Marriage Problem

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1 Formulation

Let $\mathcal{W}$ be a set of n women and $\mathcal{M}$ be a set of n men. Let $\mathcal{P}$ be the set of all possible ordered pairs (w, m), where $w \in W$ and $m \in M$. For each woman, her preferences are given as an ordered list $m_1, m_2, \ldots, m_n$ such that m_i is ranked higher than m_{i+1}, $\forall i = 1 \ldots n$. Similarly, for each man, a list of his preferences is given in the form $w_1, w_2, \ldots, w_n$ where w_i preceeds w_{i+1} only if she is ranked higher by m. We want to find a subset $\mathcal{S}$ of $\mathcal{P}$ consisting of n pairs such that the following two conditions are satisfied.

1. Each $w \in W$ and $m \in M$ appears in exactly one of the pairs $(w, m) \in S$.
2. There doesn't exist two pairs $(w, m) \in S$ and $(w', m') \in S$ such that m' is ranked higher than m by w, and w is ranked higher than w' by m'.

Such a subset S is called a *stable marriage* or a *stable matching*.

2 Algorithm

In the following version of the algorithm, the women are proposing.

Function $\mathcal{S}$ = `stableMarriage`($\mathcal{W}, \mathcal{M}$, womenPrefLists, menPrefLists)

Initially all women and men are `FREE`, and $\mathcal{S}$ is `EMPTY`
while there is w who is `FREE` and hasn't proposed to every man m **do**
  $m \leftarrow$ the next highest ranked man in w's preference list to whom she didn't yet propose
  if m is `FREE` **then**
    add (w, m) to $\mathcal{S}$
    $w \leftarrow$ `NOT_FREE`
    $m \leftarrow$ `NOT_FREE`
  else
    /* m is matched with w' */
    if w is ranked higher than w' by m **then**
      Remove (w', m) from $\mathcal{S}$
      $w' \leftarrow$ `FREE`
      add (w, m) to $\mathcal{S}$
      $w \leftarrow$ `NOT_FREE`
    end if
  end if
end while
return $\mathcal{S}$

References

[1] Jon Kleinberg and Éva Tardos. *Algorithm Analysis*. Addison Wesley, 2006.